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Boolean Algebra

Worked Solutions

Boolean laws, simplification and proofs.

GCSE & A-Level Computer Science

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Boolean Algebra

150 practice questions - laws, truth tables, De Morgan's, simplification, proofs

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Section A: Apply the Law (20 marks)

Simplify each expression and state the Boolean law used.

Q	Expression	Simplified to	Law Used
1	$A \text{ AND } 1$	A	Identity Law
2	$A \text{ OR } 0$	A	Identity Law
3	$A \text{ AND } 0$	0	Null Law
4	$A \text{ OR } 1$	1	Null Law
5	$A \text{ AND } A$	A	Idempotent Law
6	$A \text{ OR } A$	A	Idempotent Law
7	$A \text{ AND NOT}(A)$	0	Complement Law
8	$A \text{ OR NOT}(A)$	1	Complement Law
9	$\text{NOT}(\text{NOT}(A))$	A	Double Negation
10	$A \text{ AND } B$	$B \text{ AND } A$	Commutative Law
11	$A \text{ OR } (B \text{ OR } C)$	$(A \text{ OR } B) \text{ OR } C$	Associative Law
12	$A \text{ AND } (A \text{ OR } B)$	A	Absorption Law
13	$A \text{ OR } (A \text{ AND } B)$	A	Absorption Law
14	$\text{NOT}(A \text{ AND } B)$	$\text{NOT}(A) \text{ OR } \text{NOT}(B)$	De Morgan's Law
15	$\text{NOT}(A \text{ OR } B)$	$\text{NOT}(A) \text{ AND } \text{NOT}(B)$	De Morgan's Law
16	$A \text{ AND } (B \text{ OR } C)$	$(A \text{ AND } B) \text{ OR } (A \text{ AND } C)$	Distributive Law
17	$(A \text{ OR } B)(A \text{ OR } C)$	$A \text{ OR } (B \text{ AND } C)$	Distributive Law
18	$A \text{ AND } 1 \text{ AND } A$	A	Identity + Idempotent
19	$\text{NOT}(\text{NOT}(A) \text{ OR } \text{NOT}(B))$	$A \text{ AND } B$	De Morgan's + Double Negation
20	$(A \text{ AND } B) \text{ OR } (A \text{ AND } \text{NOT}(B))$	A	Complement + Identity

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Section B: Truth Table Completion (20 marks)

Complete the output column for each Boolean expression.

Method - Truth Tables:

List every input combination (use binary counting order).

Evaluate the expression for each row, recording 0 or 1 in the output column.

For 2 vars: 4 rows. For 3 vars: 8 rows. For 4 vars: 16 rows.

21. $A \text{ XOR } B$

A	B	$A \text{ XOR } B$
0	0	0
0	1	1
1	0	1
1	1	0

22. $\text{NOT}(A) \text{ XOR } B$

A	B	$\text{NOT}(A) \text{ XOR } B$
0	0	1
0	1	0
1	0	0
1	1	1

23. $\text{NOT}(A \text{ XOR } B)$

A	B	$\text{NOT}(A \text{ XOR } B)$
0	0	1
0	1	0
1	0	0
1	1	1

24. $A \text{ XNOR } \text{NOT}(B)$

A	B	$A \text{ XNOR } \text{NOT}(B)$
0	0	0
0	1	1
1	0	1
1	1	0

25. (A AND B) OR C

A	B	C	(A AND B) OR C
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

26. A AND (B OR C)

A	B	C	A AND (B OR C)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

27. NOT(A) AND B OR C

A	B	C	NOT(A) AND B OR C
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

28. NOT(A OR B) OR C

A	B	C	NOT(A OR B) OR C
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1

1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

29. A XOR (B AND C)

A	B	C	A XOR (B AND C)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

30. NOT(A) AND NOT(B) OR C

A	B	C	NOT(A) AND NOT(B) OR C
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

31. (A OR B) AND (B OR C)

A	B	C	(A OR B) AND (B OR C)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

32. A NAND (B OR C)

A	B	C	A NAND (B OR C)
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

33. (A AND B) OR (C AND D)

A	B	C	D	(A AND B) OR (C AND D)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

34. NOT(A OR B) AND (C OR D)

A	B	C	D	NOT(A OR B) AND (C OR D)
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0

0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

35. (A XOR B) AND (C XOR D)

A	B	C	D	(A XOR B) AND (C XOR D)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

36. A AND B AND C AND D

A	B	C	D	A AND B AND C AND D
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0

0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

37. NOT(A AND B) OR (C AND D)

A	B	C	D	NOT(A AND B) OR (C AND D)
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

38. (A OR C) AND (B OR D)

A	B	C	D	(A OR C) AND (B OR D)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1

0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

39. NOT(A) AND B AND NOT(C) AND D

A	B	C	D	NOT(A) AND B AND NOT(C) AND D
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

40. A XOR B XOR C XOR D

A	B	C	D	A XOR B XOR C XOR D
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0

0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

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Section C: De Morgan's Law (20 marks)

Apply De Morgan's Law to rewrite each expression in its equivalent form.

Method - De Morgan's Laws:

$\text{NOT}(A \text{ AND } B) = \text{NOT}(A) \text{ OR } \text{NOT}(B)$ (change AND to OR, negate each term)

$\text{NOT}(A \text{ OR } B) = \text{NOT}(A) \text{ AND } \text{NOT}(B)$ (change OR to AND, negate each term)

Apply repeatedly for nested expressions. Double negation cancels: $\text{NOT}(\text{NOT}(X))=X$

Q	Expression	Equivalent Form
41	$\text{NOT}(A \text{ AND } B)$	$\text{NOT}(A) \text{ OR } \text{NOT}(B)$
42	$\text{NOT}(A \text{ OR } B)$	$\text{NOT}(A) \text{ AND } \text{NOT}(B)$
43	$\text{NOT}(A \text{ AND } B \text{ AND } C)$	$\text{NOT}(A) \text{ OR } \text{NOT}(B) \text{ OR } \text{NOT}(C)$
44	$\text{NOT}(A \text{ OR } B \text{ OR } C)$	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C)$
45	$\text{NOT}(A) \text{ AND } \text{NOT}(B)$	$\text{NOT}(A \text{ OR } B)$
46	$\text{NOT}(A) \text{ OR } \text{NOT}(B)$	$\text{NOT}(A \text{ AND } B)$
47	$\text{NOT}((A \text{ AND } B) \text{ OR } C)$	$(\text{NOT}(A) \text{ OR } \text{NOT}(B)) \text{ AND } \text{NOT}(C)$
48	$\text{NOT}(A \text{ AND } (B \text{ OR } C))$	$\text{NOT}(A) \text{ OR } (\text{NOT}(B) \text{ AND } \text{NOT}(C))$
49	$\text{NOT}((A \text{ OR } B) \text{ AND } (C \text{ OR } D))$	$(\text{NOT}(A) \text{ AND } \text{NOT}(B)) \text{ OR } (\text{NOT}(C) \text{ AND } \text{NOT}(D))$
50	$\text{NOT}(\text{NOT}(A) \text{ OR } \text{NOT}(B))$	$A \text{ AND } B$
51	$\text{NOT}(\text{NOT}(A) \text{ AND } \text{NOT}(B))$	$A \text{ OR } B$
52	$\text{NOT}(\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C))$	$A \text{ OR } B \text{ OR } C$
53	$\text{NOT}(A \text{ OR } B) \text{ AND } \text{NOT}(C \text{ OR } D)$	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C) \text{ AND } \text{NOT}(D)$
54	$\text{NOT}((A \text{ XOR } B) \text{ AND } C)$	$\text{NOT}(A \text{ XOR } B) \text{ OR } \text{NOT}(C)$
55	$\text{NOT}(A \text{ NAND } B)$	$A \text{ AND } B$
56	$\text{NOT}(A \text{ NOR } B)$	$A \text{ OR } B$
57	$\text{NOT}(A \text{ OR } B \text{ OR } C \text{ OR } D)$	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C) \text{ AND } \text{NOT}(D)$
58	$\text{NOT}((A \text{ AND } B) \text{ AND } (C \text{ AND } D))$	$\text{NOT}(A \text{ AND } B) \text{ OR } \text{NOT}(C \text{ AND } D)$
59	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C)$	$\text{NOT}(A \text{ OR } B \text{ OR } C)$
60	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C) \text{ AND } \text{NOT}(D)$	$\text{NOT}(A \text{ OR } B \text{ OR } C \text{ OR } D)$

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Section D: Full Simplification (30 marks)

Fully simplify each Boolean expression. Show each step and state the law used.

Method - Simplification Strategy:

Identify patterns: double negation, complement pairs ($A \text{ AND } \text{NOT}(A)=0$), idempotent ($A \text{ AND } A=A$).

Try factoring before distributing. Look for absorption: $A \text{ AND } (A \text{ OR } B) = A$.

Apply De Morgan's to move NOT inside brackets.

Q	Expression	Result	Step-by-Step Working
61	$A \text{ AND } A \text{ AND } B$	$A \text{ AND } B$	<ol style="list-style-type: none"> $A \text{ AND } A = A$ (Idempotent Law) Result: $A \text{ AND } B$
62	$A \text{ OR } A \text{ OR } B$	$A \text{ OR } B$	<ol style="list-style-type: none"> $A \text{ OR } A = A$ (Idempotent Law) Result: $A \text{ OR } B$
63	$A \text{ AND } (A \text{ OR } B)$	A	<ol style="list-style-type: none"> Absorption Law: $A \text{ AND } (A \text{ OR } B) = A$ directly
64	$A \text{ OR } (A \text{ AND } B)$	A	<ol style="list-style-type: none"> Absorption Law: $A \text{ OR } (A \text{ AND } B) = A$ directly
65	$(A \text{ AND } B) \text{ OR } (A \text{ AND } \text{NOT}(B))$	A	<ol style="list-style-type: none"> Factor: $A \text{ AND } (B \text{ OR } \text{NOT}(B))$ $B \text{ OR } \text{NOT}(B) = 1$ (Complement Law) $A \text{ AND } 1 = A$ (Identity Law)
66	$A \text{ AND } 1 \text{ OR } A \text{ AND } 0$	A	<ol style="list-style-type: none"> $A \text{ AND } 1 = A$ (Identity Law) $A \text{ AND } 0 = 0$ (Null Law) $A \text{ OR } 0 = A$ (Identity Law)
67	$\text{NOT}(\text{NOT}(A \text{ AND } B))$	$A \text{ AND } B$	<ol style="list-style-type: none"> Double Negation: $\text{NOT}(\text{NOT}(X)) = X$ where $X = A \text{ AND } B$ Result: $A \text{ AND } B$
68	$(A \text{ OR } B) \text{ AND } (A \text{ OR } \text{NOT}(B))$	A	<ol style="list-style-type: none"> Distributive: $A \text{ OR } (B \text{ AND } \text{NOT}(B))$ $B \text{ AND } \text{NOT}(B) = 0$ (Complement Law) $A \text{ OR } 0 = A$ (Identity Law)
69	$A \text{ AND } B \text{ AND } 1 \text{ AND } A$	$A \text{ AND } B$	<ol style="list-style-type: none"> $A \text{ AND } 1 = A$ (Identity Law) $A \text{ AND } A = A$ (Idempotent Law) Result: $A \text{ AND } B$
70	$(A \text{ AND } \text{NOT}(A)) \text{ OR } B$	B	<ol style="list-style-type: none"> $A \text{ AND } \text{NOT}(A) = 0$ (Complement Law) $0 \text{ OR } B = B$ (Identity Law)
71	$\text{NOT}(A) \text{ AND } (A \text{ OR } B)$	$\text{NOT}(A) \text{ AND } B$	<ol style="list-style-type: none"> Distribute: $(\text{NOT}(A) \text{ AND } A) \text{ OR } (\text{NOT}(A) \text{ AND } B)$ $\text{NOT}(A) \text{ AND } A = 0$ (Complement Law) $0 \text{ OR } (\text{NOT}(A) \text{ AND } B) = \text{NOT}(A) \text{ AND } B$
72	$A \text{ OR } (\text{NOT}(A) \text{ AND } B)$	$A \text{ OR } B$	<ol style="list-style-type: none"> Distribute OR: $(A \text{ OR } \text{NOT}(A)) \text{ AND } (A \text{ OR } B)$ $A \text{ OR } \text{NOT}(A) = 1$ (Complement Law) $1 \text{ AND } (A \text{ OR } B) = A \text{ OR } B$

73	$(A \text{ OR } B) \text{ AND } (\text{NOT}(A) \text{ OR } B)$	B	<ol style="list-style-type: none"> 1. Distribute: $B \text{ OR } (A \text{ AND } \text{NOT}(A))$ 2. $A \text{ AND } \text{NOT}(A) = 0$ 3. $B \text{ OR } 0 = B$
74	$\text{NOT}(\text{NOT}(A) \text{ OR } \text{NOT}(B))$	$A \text{ AND } B$	<ol style="list-style-type: none"> 1. De Morgan's on $\text{NOT}(A) \text{ OR } \text{NOT}(B)$: $\text{NOT}(\text{NOT}(A) \text{ OR } \text{NOT}(B)) = \text{NOT}(\text{NOT}(A) \text{ AND } \text{NOT}(B))$ 2. Double Negation: $A \text{ AND } B$
75	$(A \text{ AND } B) \text{ OR } (\text{NOT}(A) \text{ AND } B)$	B	<ol style="list-style-type: none"> 1. Factor B: $B \text{ AND } (A \text{ OR } \text{NOT}(A))$ 2. $A \text{ OR } \text{NOT}(A) = 1$ 3. $B \text{ AND } 1 = B$
76	$\text{NOT}(A \text{ AND } B) \text{ AND } B$	$\text{NOT}(A) \text{ AND } B$	<ol style="list-style-type: none"> 1. $\text{NOT}(A \text{ AND } B) = \text{NOT}(A) \text{ OR } \text{NOT}(B)$ (De Morgan's) 2. $(\text{NOT}(A) \text{ OR } \text{NOT}(B)) \text{ AND } B$ 3. Distribute: $(\text{NOT}(A) \text{ AND } B) \text{ OR } (\text{NOT}(B) \text{ AND } B)$ 4. $\text{NOT}(B) \text{ AND } B = 0 \rightarrow \text{NOT}(A) \text{ AND } B$
77	$A \text{ AND } (\text{NOT}(A) \text{ OR } B)$	$A \text{ AND } B$	<ol style="list-style-type: none"> 1. Distribute: $(A \text{ AND } \text{NOT}(A)) \text{ OR } (A \text{ AND } B)$ 2. $A \text{ AND } \text{NOT}(A) = 0$ 3. $0 \text{ OR } (A \text{ AND } B) = A \text{ AND } B$
78	$(A \text{ AND } B) \text{ OR } (A \text{ AND } B \text{ AND } C)$	$A \text{ AND } B$	<ol style="list-style-type: none"> 1. Factor: $A \text{ AND } B \text{ AND } (1 \text{ OR } C)$ 2. $1 \text{ OR } C = 1$ (Null Law) 3. $A \text{ AND } B \text{ AND } 1 = A \text{ AND } B$
79	$\text{NOT}(A \text{ OR } B) \text{ OR } (\text{NOT}(A) \text{ AND } B)$	$\text{NOT}(A)$	<ol style="list-style-type: none"> 1. $\text{NOT}(A \text{ OR } B) = \text{NOT}(A) \text{ AND } \text{NOT}(B)$ (De Morgan's) 2. $\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ OR } \text{NOT}(A) \text{ AND } B$ 3. Factor $\text{NOT}(A)$: $\text{NOT}(A) \text{ AND } (\text{NOT}(B) \text{ OR } B)$ 4. $\text{NOT}(B) \text{ OR } B = 1 \rightarrow \text{NOT}(A)$
80	$A \text{ AND } B \text{ OR } \text{NOT}(A) \text{ AND } B$	B	<ol style="list-style-type: none"> 1. Factor B: $B \text{ AND } (A \text{ OR } \text{NOT}(A))$ 2. $A \text{ OR } \text{NOT}(A) = 1 \rightarrow B$
81	$(A \text{ OR } \text{NOT}(A)) \text{ AND } B$	B	<ol style="list-style-type: none"> 1. $A \text{ OR } \text{NOT}(A) = 1$ (Complement Law) 2. $1 \text{ AND } B = B$ (Identity Law)
82	$\text{NOT}(A) \text{ OR } A$	1	1. Complement Law: $A \text{ OR } \text{NOT}(A) = 1$ directly
83	$A \text{ AND } B \text{ AND } \text{NOT}(A \text{ OR } B)$	0	<ol style="list-style-type: none"> 1. $\text{NOT}(A \text{ OR } B) = \text{NOT}(A) \text{ AND } \text{NOT}(B)$ (De Morgan's) 2. $A \text{ AND } B \text{ AND } \text{NOT}(A) \text{ AND } \text{NOT}(B)$ 3. $A \text{ AND } \text{NOT}(A) = 0 \rightarrow \text{result is } 0$
84	$(A \text{ AND } B) \text{ OR } \text{NOT}(A \text{ AND } B)$	1	<ol style="list-style-type: none"> 1. $X \text{ OR } \text{NOT}(X) = 1$ (Complement Law) where $X = A \text{ AND } B$ 2. Result: 1
85	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } (A \text{ OR } B)$	0	<ol style="list-style-type: none"> 1. $\text{NOT}(A) \text{ AND } \text{NOT}(B) = \text{NOT}(A \text{ OR } B)$ (De Morgan's) 2. $\text{NOT}(A \text{ OR } B) \text{ AND } (A \text{ OR } B) = 0$ (Complement Law)
86	$(A \text{ OR } B) \text{ AND } \text{NOT}(A) \text{ AND } \text{NOT}(B)$	0	<ol style="list-style-type: none"> 1. $\text{NOT}(A) \text{ AND } \text{NOT}(B) = \text{NOT}(A \text{ OR } B)$ (De Morgan's) 2. $(A \text{ OR } B) \text{ AND } \text{NOT}(A \text{ OR } B) = 0$ (Complement Law)
87	$A \text{ AND } B \text{ OR } \text{NOT}(A) \text{ AND } \text{NOT}(B)$	$\text{NOT}(A \text{ XOR } B)$ (XNOR)	<ol style="list-style-type: none"> 1. This is the definition of XNOR: output 1 when $A=B$ 2. Also written as $\text{NOT}(A \text{ XOR } B)$
88	$\text{NOT}(A) \text{ AND } (A \text{ OR } B \text{ OR } C)$	$\text{NOT}(A) \text{ AND } (B \text{ OR } C)$	<ol style="list-style-type: none"> 1. Distribute: $(\text{NOT}(A) \text{ AND } A) \text{ OR } (\text{NOT}(A) \text{ AND } B) \text{ OR } (\text{NOT}(A) \text{ AND } C)$ 2. $\text{NOT}(A) \text{ AND } A = 0$ (Complement Law) 3. $0 \text{ OR } \text{NOT}(A) \text{ AND } B \text{ OR } \text{NOT}(A) \text{ AND } C = \text{NOT}(A) \text{ AND } (B \text{ OR } C)$
89	$\text{NOT}(A) \text{ AND } B \text{ OR } A \text{ AND } \text{NOT}(B)$	$A \text{ XOR } B$	<ol style="list-style-type: none"> 1. $\text{NOT}(A) \text{ AND } B$: output 1 when $A=0, B=1$ 2. $A \text{ AND } \text{NOT}(B)$: output 1 when $A=1, B=0$ 3. Together: output 1 when A differs from $B = A \text{ XOR } B$

90	NOT(A XOR B)	A XNOR B = (A AND B) OR (NOT(A) AND NOT(B))	<p>XOR is true when A ≠ B. NOT(XOR) is true when A = B</p> <p>2. Expanded: (A AND B) OR (NOT(A) AND NOT(B))</p>
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Boolean Algebra

Section E: Prove the Equivalence (20 marks)

Show step-by-step that each pair of expressions is equivalent.

Method - Proof Strategy:

Start from one side (usually the more complex) and transform it step by step.

State the law applied at each step.

End with the expression on the other side of the equation.

Q	Statement to Prove	Proof / Your Working
9 1	Show $A \text{ AND } (B \text{ OR } C) = (A \text{ AND } B) \text{ OR } (A \text{ AND } C)$	<i>Distributive Law applied directly. LHS = RHS. QED</i>
9 2	Show $\text{NOT}(\text{NOT}(A)) = A$	<i>Double Negation Law: negating twice restores the original. QED</i>
9 3	Show $(A \text{ AND } B) \text{ OR } A = A$	<i>Reorder: $A \text{ OR } (A \text{ AND } B) = A$ by Absorption Law. QED</i>
9 4	Show $A \text{ AND } (A \text{ OR } B) = A$	<i>Absorption Law applied directly. QED</i>
9 5	Show $\text{NOT}(A) \text{ AND } (A \text{ OR } B) = \text{NOT}(A) \text{ AND } B$	<i>Distribute: $(\text{NOT}(A) \text{ AND } A) \text{ OR } (\text{NOT}(A) \text{ AND } B) \rightarrow 0 \text{ OR } \text{NOT}(A)B \rightarrow \text{NOT}(A) \text{ AND } B$</i>
9 6	Show $(A \text{ OR } B) \text{ AND } (A \text{ OR } \text{NOT}(B)) = A$	<i>Distribute: $A \text{ OR } (B \text{ AND } \text{NOT}(B)) = A \text{ OR } 0 = A$</i>
9 7	Show $\text{NOT}(A \text{ AND } B) = \text{NOT}(A) \text{ OR } \text{NOT}(B)$	<i>De Morgan's First Law - this is the definition. QED</i>
9 8	Show $\text{NOT}(A \text{ OR } B) = \text{NOT}(A) \text{ AND } \text{NOT}(B)$	<i>De Morgan's Second Law - this is the definition. QED</i>
9 9	Show $A \text{ OR } (\text{NOT}(A) \text{ AND } B) = A \text{ OR } B$	<i>$(A \text{ OR } \text{NOT}(A)) \text{ AND } (A \text{ OR } B) = 1 \text{ AND } (A \text{ OR } B) = A \text{ OR } B$ [Distributive + Complement]</i>
1 0 0	Show $(A \text{ XOR } B) = (A \text{ OR } B) \text{ AND } \text{NOT}(A \text{ AND } B)$	<i>$A \text{ XOR } B = (A \text{ AND } \text{NOT}(B)) \text{ OR } (\text{NOT}(A) \text{ AND } B) = \text{same as } (A \text{ OR } B) \text{ AND } \text{NOT}(A \text{ AND } B)$. QED</i>
1 0 1	Show $\text{NOT}(A) \text{ AND } \text{NOT}(B) = \text{NOT}(A \text{ OR } B)$	<i>De Morgan's: $\text{NOT}(A \text{ OR } B) = \text{NOT}(A) \text{ AND } \text{NOT}(B)$. QED</i>
1 0 2	Show $(A \text{ AND } B) \text{ OR } (A \text{ AND } B \text{ AND } C) = A \text{ AND } B$	<i>Factor: $A \text{ AND } B \text{ AND } (1 \text{ OR } C) = A \text{ AND } B \text{ AND } 1 = A \text{ AND } B$</i>

1 0 3	Show $A \text{ OR } (A \text{ AND NOT}(B)) = A$	<i>Factor: $A \text{ AND } (1 \text{ OR NOT}(B)) = A \text{ AND } 1 = A$</i>
1 0 4	Show $(A \text{ AND NOT}(B)) \text{ OR } (\text{NOT}(A) \text{ AND } B) \text{ OR } (A \text{ AND } B) = A \text{ OR } B$	<i>First two = $A \text{ XOR } B$. Adding $A \text{ AND } B$: $\text{XOR OR AND} = \text{OR}$</i>
1 0 5	Show $\text{NOT}(A) \text{ AND } (A \text{ OR } B) \text{ AND } (A \text{ OR } C) = \text{NOT}(A) \text{ AND } B \text{ AND } C$	<i>$\text{NOT}(A) \text{ AND } (A \text{ OR } B) = \text{NOT}(A) \text{ AND } B$. Then $\text{AND } (A \text{ OR } C)$: $\text{NOT}(A) \text{ AND } B \text{ AND } C$</i>
1 0 6	Show $A \text{ AND } B \text{ AND } C = \text{NOT}(\text{NOT}(A) \text{ OR } \text{NOT}(B) \text{ OR } \text{NOT}(C))$	<i>De Morgan's: $\text{NOT}(\text{NOT}(A) \text{ OR } \text{NOT}(B) \text{ OR } \text{NOT}(C)) = A \text{ AND } B \text{ AND } C$</i>
1 0 7	Show $(A \text{ OR } B) \text{ AND } (\text{NOT}(A) \text{ OR } C) \text{ AND } (B \text{ OR } C) = (A \text{ OR } B) \text{ AND } (\text{NOT}(A) \text{ OR } C)$	<i>Consensus Theorem: the third factor is redundant.</i>
1 0 8	Show $\text{NOT}(A \text{ AND } B) \text{ AND } A = \text{NOT}(B) \text{ AND } A$	<i>$(\text{NOT}(A) \text{ OR } \text{NOT}(B)) \text{ AND } A = (\text{NOT}(B) \text{ AND } A)$ since $\text{NOT}(A) \text{ AND } A = 0$</i>
1 0 9	Show $A \text{ NAND } A = \text{NOT}(A)$	<i>$A \text{ NAND } A = \text{NOT}(A \text{ AND } A) = \text{NOT}(A)$ by Idempotent Law</i>
1 1 0	Show $A \text{ NOR } A = \text{NOT}(A)$	<i>$A \text{ NOR } A = \text{NOT}(A \text{ OR } A) = \text{NOT}(A)$ by Idempotent Law</i>

Boolean Algebra

SOLUTIONS

SOLUTIONS

Section F: Derive the Expression (20 marks)

Write a Boolean expression that satisfies each description.

Method - Derive Expression:

Identify which inputs must be 1, which must be 0.

AND for 'all must be true'. OR for 'any must be true'.

NOT to invert an input. XOR for 'exactly one different'.

Q	Description	Boolean Expression
1 1 1	Output is 1 only when A=1 AND B=1	A AND B
1 1 2	Output is 1 when A=0 AND B=0	NOT(A) AND NOT(B)
1 1 3	Output is 0 only when A=0 AND B=0	A OR B
1 1 4	Output is 0 only when A=1 AND B=1	NOT(A AND B) = NAND
1 1 5	Output is 1 when A and B are different	A XOR B
1 1 6	Output is 1 when A and B are the same	NOT(A XOR B) = XNOR
1 1 7	Output is 1 when at least one of A,B,C is 1	A OR B OR C
1 1 8	Output is 1 only when all of A,B,C are 1	A AND B AND C
1 1 9	Output is 1 when exactly one of A,B is 1	A XOR B
1 2 0	Output is 1 when A=1 and B=0	A AND NOT(B)
1 2 1	Output is 1 when the number of 1s in A,B,C is odd	A XOR B XOR C

1 2 2	Output is 0 when any input is 1	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C) = \text{NOR}(A,B,C)$
1 2 3	Output is 1 unless all inputs are 1	$\text{NOT}(A \text{ AND } B \text{ AND } C) = \text{NAND}(A,B,C)$
1 2 4	Output is 1 when A=1 OR when B=1 AND C=1	$A \text{ OR } (B \text{ AND } C)$
1 2 5	Output is 1 when A=0 AND B=0 AND C=0	$\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ AND } \text{NOT}(C)$
1 2 6	Output is 1 when A differs from B, and C=1	$(A \text{ XOR } B) \text{ AND } C$
1 2 7	Output equals A when B=1; 0 when B=0	$A \text{ AND } B$
1 2 8	Output is 1 when A=B AND C=1	$\text{NOT}(A \text{ XOR } B) \text{ AND } C$
1 2 9	Output is NOT(A) when B=1; 1 when B=0	$\text{NOT}(A) \text{ OR } \text{NOT}(B)$
1 3 0	Output is 1 when at least two of A,B,C are 1	$(A \text{ AND } B) \text{ OR } (A \text{ AND } C) \text{ OR } (B \text{ AND } C)$

Boolean Algebra

Section G: Mixed Challenge (20 marks)

Fully simplify each expression. Multiple laws may be required.

Method - Mixed Challenge Tips:

Look for De Morgan's opportunities first (NOT over AND/OR).

Use XOR identities: $A \text{ XOR } 0 = A$, $A \text{ XOR } 1 = \text{NOT}(A)$, $A \text{ XOR } A = 0$.

Factor common sub-expressions then apply absorption or complement.

Q	Expression	Result	Step-by-Step Working
1 3 1	$\text{NOT}(A \text{ OR } B) \text{ OR } (A \text{ AND } \text{NOT}(B))$	$\text{NOT}(B)$	<ol style="list-style-type: none"> $\text{NOT}(A \text{ OR } B) = \text{NOT}(A) \text{ AND } \text{NOT}(B)$ (De Morgan's) $\text{NOT}(A) \text{ AND } \text{NOT}(B) \text{ OR } A \text{ AND } \text{NOT}(B) \rightarrow \text{NOT}(B) \text{ AND } (\text{NOT}(A) \text{ OR } A)$ $\text{NOT}(A) \text{ OR } A = 1 \rightarrow \text{NOT}(B)$
1 3 2	$(A \text{ OR } B) \text{ AND } (\text{NOT}(A) \text{ OR } \text{NOT}(B))$	$A \text{ XOR } B$	<ol style="list-style-type: none"> $\text{NOT}(A) \text{ OR } \text{NOT}(B) = \text{NOT}(A \text{ AND } B)$ (De Morgan's) $(A \text{ OR } B) \text{ AND } \text{NOT}(A \text{ AND } B) = \text{XOR}$ by definition
1 3 3	$A \text{ AND } B \text{ OR } \text{NOT}(A) \text{ AND } \text{NOT}(B)$	$\text{NOT}(A \text{ XOR } B)$ (XNOR)	<ol style="list-style-type: none"> Output 1 when $A=B$ (both 0 or both 1) - this is XNOR by definition
1 3 4	$\text{NOT}(\text{NOT}(A) \text{ AND } B) \text{ AND } \text{NOT}(A \text{ AND } \text{NOT}(B))$	$\text{NOT}(A \text{ XOR } B)$	<ol style="list-style-type: none"> $\text{NOT}(\text{NOT}(A) \text{ AND } B) = A \text{ OR } \text{NOT}(B)$ (De Morgan's) $\text{NOT}(A \text{ AND } \text{NOT}(B)) = \text{NOT}(A) \text{ OR } B$ (De Morgan's) $(A \text{ OR } \text{NOT}(B)) \text{ AND } (\text{NOT}(A) \text{ OR } B) = \text{NOT}(A \text{ XOR } B)$
1 3 5	$(A \text{ AND } B \text{ AND } C) \text{ OR } (\text{NOT}(A) \text{ AND } B \text{ AND } C) \text{ OR } \text{NOT}(B \text{ AND } C)$	1	<ol style="list-style-type: none"> First two terms: $B \text{ AND } C \text{ AND } (A \text{ OR } \text{NOT}(A)) = B \text{ AND } C$ $B \text{ AND } C \text{ OR } \text{NOT}(B \text{ AND } C) = 1$ (Complement Law)
1 3 6	$\text{NOT}(A \text{ NAND } B)$	$A \text{ AND } B$	<ol style="list-style-type: none"> $\text{NOT}(A \text{ NAND } B) = \text{NOT}(\text{NOT}(A \text{ AND } B)) = A \text{ AND } B$ (Double Negation)
1 3 7	$\text{NOT}(A \text{ NOR } B)$	$A \text{ OR } B$	<ol style="list-style-type: none"> $\text{NOT}(A \text{ NOR } B) = \text{NOT}(\text{NOT}(A \text{ OR } B)) = A \text{ OR } B$ (Double Negation)
1 3 8	$A \text{ XOR } 0$	A	<ol style="list-style-type: none"> XOR with 0: if $A=0$, $0 \text{ XOR } 0=0$; if $A=1$, $1 \text{ XOR } 0=1 \rightarrow \text{output} = A$
1 3 9	$A \text{ XOR } 1$	$\text{NOT}(A)$	<ol style="list-style-type: none"> XOR with 1: if $A=0$, $0 \text{ XOR } 1=1$; if $A=1$, $1 \text{ XOR } 1=0 \rightarrow \text{output} = \text{NOT}(A)$
1 4 0	$A \text{ XOR } A$	0	<ol style="list-style-type: none"> Same input XORed: always 0 (Complement-like identity for XOR)
1 4 1	$A \text{ XNOR } A$	1	<ol style="list-style-type: none"> Same input XNORed: always 1 (if $A=A$ they are always equal)

1 4 2	$(A \text{ AND } B) \text{ XOR } (A \text{ AND } B)$	0	1. $X \text{ XOR } X = 0$ where $X = A \text{ AND } B$
1 4 3	$A \text{ AND NOT}(A \text{ XOR } B)$	$A \text{ AND } B$	1. $\text{NOT}(A \text{ XOR } B) = \text{XNOR} = 1$ when $A=B$ 2. $A \text{ AND XNOR}$: only 1 when $A=1 \text{ AND } A=B \rightarrow A \text{ AND } B$
1 4 4	$\text{NOT}(A \text{ AND NOT}(B)) \text{ AND NOT}(B)$	$\text{NOT}(A) \text{ AND NOT}(B)$	1. $\text{NOT}(A \text{ AND NOT}(B)) = \text{NOT}(A) \text{ OR } B$ (De Morgan's) 2. $(\text{NOT}(A) \text{ OR } B) \text{ AND NOT}(B) = (\text{NOT}(A) \text{ AND NOT}(B)) \text{ OR } (B \text{ AND NOT}(B))$ 3. $B \text{ AND NOT}(B) = 0 \rightarrow \text{NOT}(A) \text{ AND NOT}(B)$
1 4 5	$(A \text{ AND } B) \text{ OR } (\text{NOT}(A) \text{ AND } B) \text{ OR } (A \text{ AND NOT}(B))$	$A \text{ OR } B$	1. $(A \text{ AND } B) \text{ OR } (\text{NOT}(A) \text{ AND } B) = B$ (factor B) 2. $B \text{ OR } (A \text{ AND NOT}(B)) = B \text{ OR } A$ (absorption variant) = $A \text{ OR } B$
1 4 6	$\text{NOT}(A) \text{ OR } (A \text{ AND } B)$	$\text{NOT}(A) \text{ OR } B$	1. $\text{NOT}(A) \text{ OR } (A \text{ AND } B) = (\text{NOT}(A) \text{ OR } A) \text{ AND } (\text{NOT}(A) \text{ OR } B) = 1 \text{ AND } (\text{NOT}(A) \text{ OR } B)$
1 4 7	$(A \text{ OR } B) \text{ AND NOT}(\text{NOT}(A) \text{ AND NOT}(B))$	$A \text{ OR } B$	1. $\text{NOT}(\text{NOT}(A) \text{ AND NOT}(B)) = \text{NOT}(\text{NOT}(A \text{ OR } B)) = A \text{ OR } B$ (De Morgan's + Double Negation) 2. $(A \text{ OR } B) \text{ AND } (A \text{ OR } B) = A \text{ OR } B$ (Idempotent)
1 4 8	$\text{NOT}(A \text{ AND } B) \text{ AND NOT}(\text{NOT}(A) \text{ AND NOT}(B))$	$A \text{ XOR } B$	1. $\text{NOT}(A \text{ AND } B) = \text{NAND}$. $\text{NOT}(\text{NOT}(A) \text{ AND NOT}(B)) = A \text{ OR } B$ (De Morgan's x2) 2. $\text{NAND AND } (A \text{ OR } B) = \text{NOT}(A \text{ AND } B) \text{ AND } (A \text{ OR } B) = \text{XOR definition}$
1 4 9	$A \text{ XOR } (A \text{ AND } B)$	$A \text{ AND NOT}(B)$	1. XOR expansion: $(A \text{ AND NOT}(A \text{ AND } B)) \text{ OR } (\text{NOT}(A) \text{ AND } A \text{ AND } B)$ 2. $\text{NOT}(A) \text{ AND } A \text{ AND } B = 0 \rightarrow A \text{ AND NOT}(A \text{ AND } B)$ 3. $\text{NOT}(A \text{ AND } B)$ applied: $A \text{ AND } (\text{NOT}(A) \text{ OR NOT}(B)) = A \text{ AND NOT}(B)$
1 5 0	$(A \text{ OR } B \text{ OR } C) \text{ AND NOT}(A \text{ OR } B)$	$\text{NOT}(A) \text{ AND NOT}(B) \text{ AND } C$	1. $\text{NOT}(A \text{ OR } B) = \text{NOT}(A) \text{ AND NOT}(B)$ (De Morgan's) 2. $\text{NOT}(A) \text{ AND NOT}(B) \text{ AND } (A \text{ OR } B \text{ OR } C) \rightarrow$ distribute; A,B terms cancel 3. Result: $\text{NOT}(A) \text{ AND NOT}(B) \text{ AND } C$

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